Capacitance matrix for combline and interdigital:

$$C_{j,j+1}/\epsilon|_{j=1,N-1} = 376.7/\sqrt{\epsilon_r} \cdot y_{j,j+1}$$
$$C_1/\epsilon = 376.7/\sqrt{\epsilon_r} \cdot (Y_{a_1} - y_{12})$$
$$C_j/\epsilon|_{j=2,N-1} = 376.7/\sqrt{\epsilon_r} \cdot (Y_{a_1} - y_{j-1,j} - y_{j,j+1})$$
$$C_N/\epsilon = 376.7/\sqrt{\epsilon_r} \cdot (Y_{a_1} - y_{N-1,N}).$$

Lumped capacitance:

$$C_1^s = Y_{a_1} \cdot \cot(\theta_0)/\omega_0$$
$$y_T = Y_{a_1} - y_{12}^2/Y_{a_1}$$
$$C_c^s = Y_A^2 z_T \sin(\theta_0 - \Phi_0)/\{[(\sin\theta_0/\sin\Phi_0)^3 + (\sin\theta_0/\sin\Phi_0) \cdot Y_A^2 z_T^2 \sin^2(\theta_0 - \Phi_0)]\,\omega_0\}.$$

Combline	Interdigital
$C_{1\,\mathrm{TOTAL}}^s = C_{N\,\mathrm{TOTAL}}^s = C_1^s + C_c^s$	$C_{1\,\mathrm{TOTAL}}^s = C_{N\,\mathrm{TOTAL}}^s = C_c^s.$
$C_j^s\|_{j=2,N-1} = C_1^s$	

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On the Realizability of the Impedance Matrix for Lossy Dielectric Posts in a Rectangular Waveguide

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***Abstract* —The equivalent T network for some lossy dielectric posts in a rectangular waveguide is found to have a negative resistance in the parallel arm although the realizability conditions for the impedance matrix are strictly satisfied. Furthermore, the reactive part of the same impedance is found to be a monotonically decreasing function of frequency. These difficulties are overcome in the case of symmetrical post structures by using lattice networks. A simplified lattice network of lumped elements is developed to approximately realize the impedance matrix for resonant lossy post structures in the bandwidth.**

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I. Introduction

A method for the analysis of dielectric posts located in a rectangular waveguide has been given in [1]. In the analysis, the dielectric posts are assumed to be uniform along the narrow side of the waveguide but are otherwise of arbitrary cross section. Furthermore, the medium of the waveguide is assumed to be linear, homogeneous, isotropic, and dissipation free. The dielectric posts are likewise linear, homogeneous, isotropic, although not necessarily free from losses. Only a few of the results for circular posts free from losses and located in a rectangular waveguide whose medium is the vacuum are reported in [1]. More results for loss-free as well as lossy posts of different configurations are given in [2].

This paper addresses the question of realizability of lossy dielectric posts in a rectangular waveguide in terms of two-port networks. Of all such networks, the T network is most commonly used. This is perhaps due to the fact that it readily realizes both symmetrical and unsymmetrical impedance matrices in a very simple and straightforward fashion. However, it is found that although the realizability conditions for the impedance matrix representation of the posts are strictly satisfied, the parallel arm impedance of the corresponding T network can have a negative real part. Furthermore, for some configurations of lossy posts, the reactive part of the same impedance is found to be a monotonically decreasing function of frequency. Such a behavior is observed for some loss-free posts as well [1], [2]. This situation, although not unlikely, does rule out any chance for obtaining a lumped network representation of resonant posts valid in the bandwidth. These lumped representations are particularly useful in the design of microwave filters employing dielectric posts in a rectangular waveguide. The purpose of this paper is therefore twofold. First, it is shown that such an irregular behavior can be avoided in the case of lossy symmetrical post structures by using lattice networks. Second, it is demonstrated that a lumped lattice network can be developed to approximately realize the impedance matrix in the bandwidth. Although the latter is accomplished by means of a working example, the procedure established is believed to be general.

II. The Realizability Problem for Lossy Posts

The realizability, and symmetry whenever applicable, conditions of the computed impedance matrix of the various configurations of lossy posts considered in [1] and [2] have always been checked. In particular, the real part $R = [R_{ij}]$, $\{i, j\} = \{1,2\}$, of the impedance matrix is found to satisfy the well-known realizability conditions [3, sec. 5-11]:

$$R_{11}, R_{22} > 0$$
$$R_{11}R_{22} - R_{12}^2 > 0. \quad (1)$$

The equivalent condition that the matrix $(U - S^H S)$, where U is the identity matrix, S is the scattering matrix of the posts, and the superscript H denotes matrix Hermitian, must be positive definite for lossy passive microwave systems [4, sec. 3-3] is also always found to be satisfied. However, in attempting to realize the impedance matrix in the form of a T network, it is found that, although the realizability conditions for the impedance matrix are strictly satisfied, the resistive part of the parallel arm impedance of the T network is negative for some post configurations. Furthermore, the reactive part of the same impedance can be a monotonically decreasing function of frequency. This situa-

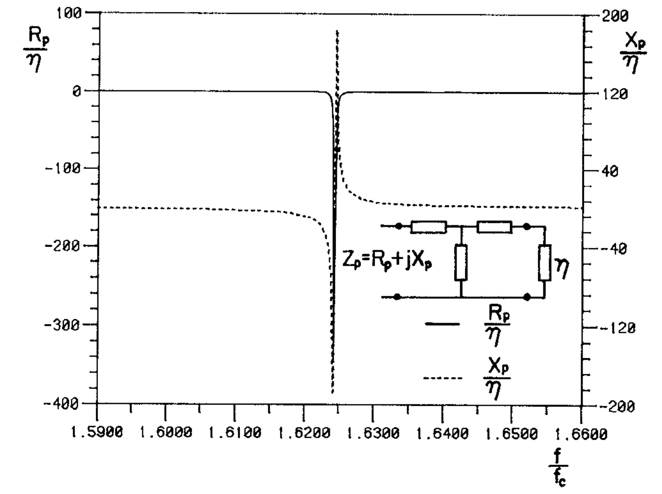

Fig 1. The change of the parallel arm impedance of the T network with frequency for a centered lossy square post of cross section area equal to that of a circular post of diameter $d = 0.15a$ and dielectric constant $\epsilon_r = 38.0 \times (1 - j10^{-4})$.

tion is encountered, for instance, in the case of centered lossy square and circular posts.

Nevertheless, as long as the realizability conditions of the impedance matrix are satisfied, a suitable two-port network can be found. As it turns out, a representation in the form of a lattice network is always found to overcome these difficulties. This is readily understood in view of Bartlett's bisection theorem for symmetrical networks [5, sec. 6-4]. Bartlett's theorem asserts that the series and cross-arm impedances of the lattice network are, respectively, identical with the input short- and open-circuit impedances of half of the original network. The branch impedances of the lattice network in the case of loss-free posts are then assured to behave in accordance with Foster's reactance theorem [4, sec. 2-2]. Similar behavior is expected in the case of lossy posts. In Fig. 1, the change of the parallel arm impedance of the T network with frequency is shown for a centered lossy square post of cross-section area equal to that of a circular post of diameter $d = 0.15a$, where a is the width of the waveguide, and dielectric constant $\epsilon_r = 38.0 \times (1 - j10^{-4})$. In Fig. 2, the change of the resistive and reactive parts of branch impedances of the lattice network with frequency is shown for the same post. In these figures, f_c and η denote, respectively, the cutoff frequency and characteristic impedance of the dominant (TE_{10}) waveguide mode. The series-arm impedance of the symmetrical lattice network is actually the same as the series-arm impedance of the symmetrical T network. It is important to recall that the loss tangent is a function of frequency. However, under the assumption of small losses, the loss tangent is kept constant throughout the frequency range of operation ($f_c < f < 2.0 f_c$).

III. The Lumped Lattice Network

The lattice network is a reasonably simple form for realizing the impedance matrix for lossy symmetrical post configurations at any fixed frequency. Network representations of waveguide discontinuities, on the other hand, cannot always be physically realized by lumped elements over the waveguide frequency range of operation. However, if the post configuration is resonant, a lattice network of lumped elements can be developed to approximately realize the impedance matrix in the bandwidth. This is demonstrated below for the centered lossy square post considered. An examination of the data obtained reveals that the resonant frequency of this post occurs at about $1.625 f_c$, the 3-dB bandwidth is $1.588 f_c \leqslant f \leqslant 1.662 f_c$, and the loaded quality factor is equal to 21.96.

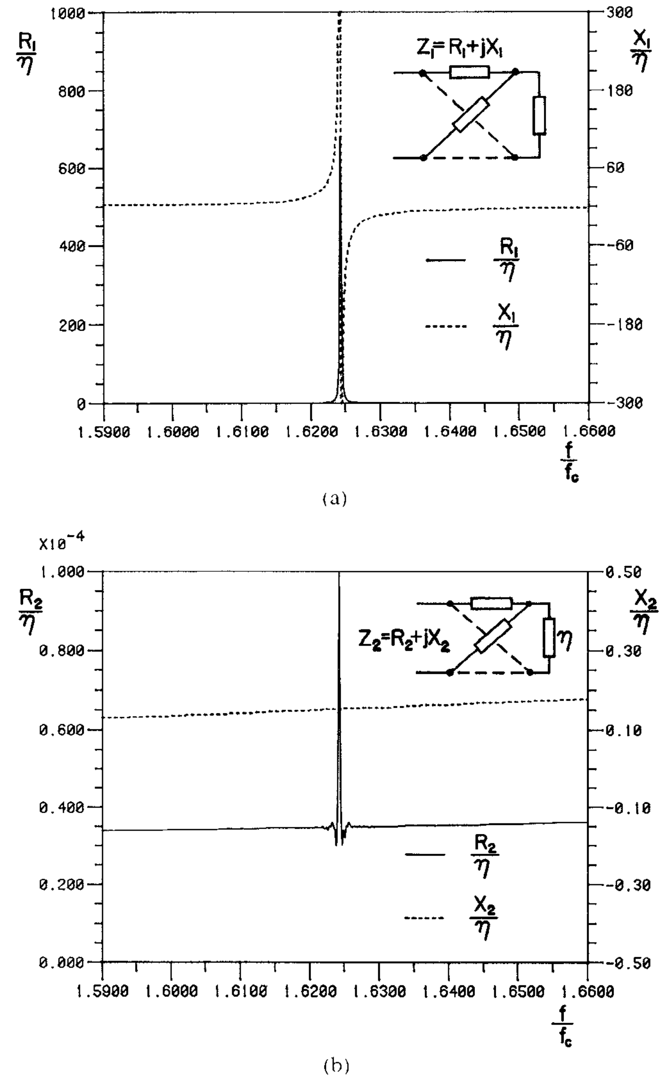

Fig. 2. The change of branch impedances of the lattice network with frequency for the centered lossy square post. Same data as in Fig. 1.

The reactive part of the series arm impedance clearly corresponds to a resonant parallel LC network. The resonant frequency of this arm is slightly off the post resonance at $f_0 \simeq 1.6243 f_c$. The resistive part, however, is impulsive in the bandwidth. Such a behavior is known to occur for reactance functions having simple poles at $s = -\alpha \pm j\omega_0$, $\omega_0 = 2\pi f_0$, in the complex s plane as α tends to zero [5, sec. 8-2]. This is in fact the situation at hand under the assumption of small losses. The impedance of this arm can then be written as

$$Z_1 \simeq \frac{2A}{\pi} \frac{s}{s^2 + \omega_0^2} \tag{2}$$

where

$$\omega_0^2 = \frac{1}{L_1 C_1}. \tag{3}$$

For $s = j\omega$, the resistive and reactive parts of Z_1 become

$$R_1(\omega) \simeq A u(\omega - \omega_0)$$

$$X(\omega) \simeq -\frac{2A}{\pi} \frac{\omega}{\omega^2 - \omega_0^2} \tag{4}$$

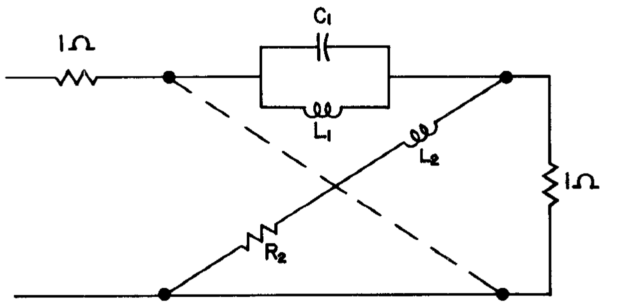

Fig. 3. The normalized lumped lattice network: $C_1 = 6.7517\omega_c^{-1}$ F, $L_1 = 0.0561\omega_c^{-1}$ H, $R_2 = 4\times10^{-5}$ Ω, and $L_2 = 0.0941\omega_c^{-1}$ H

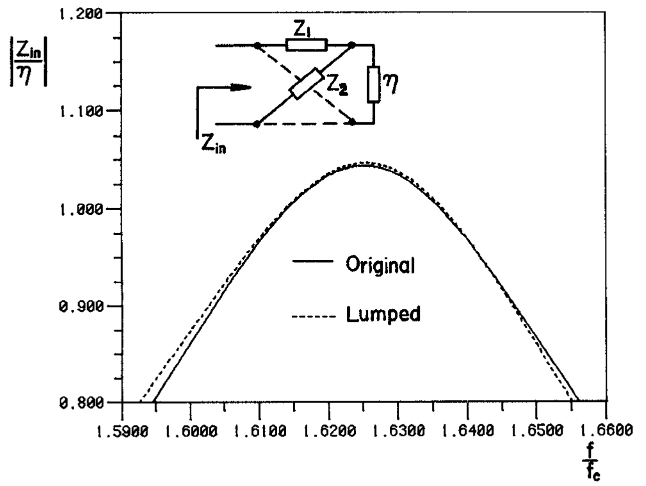

Fig. 4. The input impedance of the original network versus the input impedance of the lumped lattice network in the bandwidth.

where u is the unit impulse function, and the residue A is equal to the area under $R_1(\omega)$.

The reactance of the cross-arm impedance is clearly that of an inductor. Although sudden changes also occur in the resistive part of the cross arm, this cannot be attributed to the inductor. The value of the resistive part, however, is basically constant in the bandwidth except for this sudden change at resonance. Consequently, assuming small losses, the impedance of this arm can be realized in the bandwidth by an inductor in series with a resistor; viz.,

$$Z_2(\omega) \simeq R_2 + j\omega L_2. \tag{5}$$

The evaluation of the elements of the lumped lattice network is rather straightforward. First, the area under $R_1(\omega)$ is determined and assigned to the residue A. The capacitor C_1 is then found as

$$C_1 = \frac{\pi}{2A} \tag{6}$$

while the inductor L_1 is found from (3). Furthermore, the inductor L_2 of the cross arm is readily determined as the slope of $X_2(\omega)$, or more simply as

$$L_2 = \frac{X_2(\omega_0)}{\omega_0}. \tag{7}$$

Finally, the resistor R_2 is assigned the constant value it has in the bandwidth. The normalized lumped lattice network obtained is shown in Fig. 3. The input impedance of the developed lumped network is shown in Fig. 4, where it can be seen that it agrees rather well with the input impedance of the original network in the bandwidth. In passing, it is noted that other choices of R_2, such as $R_2(\omega_0)$ or the area under $R_2(\omega)$ in the bandwidth divided by the bandwidth, change very little of the input impedance of the lumped lattice network.

IV. Summary

It has been shown in this paper that the lattice network can be used to overcome the difficulties associated with the T network representation of symmetrical configurations of lossy dielectric posts in a rectangular waveguide. If resonant, a lattice network of lumped elements can be developed to approximately realize the impedance matrix of the posts in the bandwidth. This lumped representation is particularly useful in the design of microwave filters employing dielectric posts in a rectangular waveguide.

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Attenuation Distortion of Transient Signals in Microstrip

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Abstract —**Attenuation distortion, and combinations of dispersion and attenuation distortions, of transient signals in microstrip lines are investigated. Conduction losses are considered for the general case where the strip conductor resistivity is different from that of the ground plane. Dielectric losses are examined for commonly used isotropic substrates. Attenuation and dispersion distortions of short pulses are shown to vary as microstrip and pulse parameters are changed.**

I. Introduction

The analysis of transient signal behavior in microstrip lines is essential for the design of MIC's that operate at high switching speeds or high frequencies. This behavior has been investigated in the past only for the case of distortions due to dispersion [1]–[3]. Analyzing dispersion distortion is important because it signifi-

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